Barcan Formulas and the Limits of Possibility

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1 Background

1. Barcan Formulas

   BF. $\Diamond\exists\upsilon\varphi \supset \exists\upsilon\Diamond\varphi$

   BFC. $\exists\upsilon\Diamond\varphi \supset \Diamond\exists\upsilon\varphi$

2. "Limits"

3. Counterexamples?

   Aliens. $\Diamond\exists x@\forall y(y \neq x)$[1]

   Absentees. $\exists x\Diamond\neg\exists y(y = x)$

4. An argument for necessitism from absolute generality

   Con. $\Diamond\exists x\Diamond\neg\exists y(y = x)$

   Nec. $\Box\forall x\Box\exists y(y = x)$

2 Revisionism

1. In logic:

   - "Simple" qml (Carnap 1946; Marcus 1961): Quantification (at a world, in a model) over a common pool of *possibilia D*.
   - Kripke 1963: Frames: $\langle W, D, w_@, Q\rangle$; $Q : W \to D^n$ a *domain function*
   - "Formally elegant; philosophically unsatisfying": (Williamson 1998; Jones 2016)

2. In metaphysics:

   - Being something vs. being something *concrete* (Linsky and Zalta 1996; Williamson 1998)

     Concrete Aliens. $\Diamond\exists x(Cx \wedge @\forall y(Cy \to y \neq x)$

     Non-concrete Absentees. $\exists x(@Cx \wedge \Diamond\neg\exists y(Cy \wedge y = x))$
   - Essential vs. accidental property attribution (Hayaki 2006)
   - *Concreteness* as essential
   - Russell's "robust sense of reality"

3 Relativity

1. A puzzle for necessitists:

   i. Necessarily, Obama is something. $(\Box\exists y(y = o))$
   ii. Possibly, Obama is never conceived. $(\Diamond\neg Co)$
   iii. ∴ Possibly, Obama is something and is never conceived. (i, ii)
   $(\Diamond(\exists y(y = o) \wedge \neg Co))$

2. Context–index pragmatics

   - Kaplan (1977) and Lewis (1980) on *context* and *index*
     - "As actual" vs. "as counterfactual" (Davies and Humberstone 1980)
     - Target and source perspective
   - Context-shift and our puzzlement re: (iii)

3. Necessitism:

   (a) Nonexistence
   - Internal and external domains
   - BFC and *Absentees*

   (b) Existence
   - Simchen (2013) on BF and *Aliens*
   - Perspectival shift: closing the gap

4. Last bits

   - @-fundamentality; *w*-parity
   - RBM on mere possibilia (Marcus 1985)
   - The limits of possibility: *de dicto* and *de re*

[1] '@' an actuality-operator: $@\varphi \leftrightarrow \varphi$; $\varphi \to \Box@\varphi$, etc.

4 Relativist semantics

- *Frames*: $\langle W, D, w_@, C\rangle$; $C : W \to D^n$ a "context" function; $C(w) ::= D(c_w)$; c_w a context "centered" upon w.
- *Models*: $\langle \mathcal{F}, \mathcal{I}\rangle$, $\mathcal{F}$ a frame and $\mathcal{I}$ such that:
  - $\mathcal{I}(F^n) ::= In_{F^n} : In_{F^n}(w, v) \subseteq D(c_w)$, for $w, v \in W$
  - $\mathcal{I}(\tau) ::= In_\tau : In_\tau(w, v) \in D(c_w)$
- *Indices* on a context c_w: $\langle v, g_w\rangle$; $v \in W$ and $g_w \in D(c_w)^\omega$ a variable assignment.
- Denotation at an index $i = \langle v, g_w\rangle$, in a context c_w:

$$|\tau_k|_i^{c_w} = \begin{cases} In_{\tau_k}(w, v), \text{ when } \tau_k \text{ a constant;} \\ \\ g_w(k), \text{ when } \tau_k = x_k. \end{cases}$$

- Satisfaction at an index $i = \langle v, g_w\rangle$, in a context c_w:
  - $\vDash_i^{c_w} F^n(\tau_1, \ldots, \tau_n)$ just if $\langle |\tau_1|_i^{c_w}, \ldots, |\tau_n|_i^{c_w}\rangle \in In_{F^n}(w, v)$
  - $\vDash_i^{c_w} \tau_i = \tau_k$ just if $|\tau_i|_i^{c_w} = |\tau_k|_i^{c_w}$
  - $\vDash_i^{c_w} \neg\varphi$ just if $\nvDash_i^{c_w} \varphi$
  - $\vDash_i^{c_w} \varphi \vee \psi$ just if $\vDash_i^{c_w} \varphi$ or $\vDash_i^{c_w} \psi$ (etc.)
  - $\vDash_i^{c_w} \forall x_k \varphi$ just if for all $i' \sim_k i$, $\vDash_{i'}^{c_w} \varphi$
    $\vDash_i^{c_w} \Box\varphi$ just if for all $i' \sim_v i$, $\vDash_{i'}^{c_w} \varphi$
- Truth at a world; truth in a context; entailment, etc.:
  - At a world: $\vdash_v^{c_w} \varphi$ just if $\vDash_i^{c_w} \varphi$ when $w_i = v$
  - At a context: $\vdash^{c_w} \varphi$ just if $\vDash_{i_c}^{c_w} \varphi$; $i_c = \langle w, g_w\rangle$
  - *Simpliciter*: $\vdash \varphi$ just if $\vdash^{c_w} \varphi$ for all w
  - Entailment: $\gamma_1, \ldots, \gamma_n \vdash \varphi$ just if $\vdash \varphi$ if $\vdash \gamma_1 \wedge \ldots \wedge \gamma_n$

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